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Restricted Relational Richness and Musical Analysis*

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In recent years, I have been developing a method for analysing music of all sorts. The fruits of this method appear in a number of articles and in my book *A Theory for All Music.*[1] My method of analysis takes as one of its points of departure an approach first described in detail by Nelson Goodman in *The Structure of Appearance*[2] and thereupon elaborated in certain passages of *Meta-Variations* by the music theorist Benjamin Boretz.[3] Since my method can be construed as having a direct antecedent in philosophy, it might be of some interest to workers in that field. Because my method has found its first substantial applications in the realm of music, it might interest aestheticians. I hope that my method will be attractive to philosophers and I believe that the reasons which one can advance in its favour contain a number of philosophic novelties that are capable of further development.

The analyses that I have undertaken presuppose that there are observed phenomena to be analysed. In the case of music, I consider these phenomena to consist of sounds and scores, that is tones and notes, respectively. When I do not wish to distinguish between tones and notes, I refer to the observed phenomena simply as the "observables."

Undefined Predicates.

The second component of my analyses consists of the concepts that one employs to interpret or describe the observables. In a musical analysis, these

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concepts might be embedded in such phrases as "has pitch," "is higher (in pitch) than," "is just higher ih pitch than," "forms a pitch-interval that matches the pitch-interval formed by," "happens at a moment," "happens at a moment that is before the moment at which occurs," "is just before," "forms a time-interval that matches the time-interval formed by," "is louder than," and "has a timbre that matches the timbre of."[4] Each of these phrases corresponds to either a one-place predicate or a two-place predicate, where the places are occupied by observed tones or notes. When one or more observables occupy the places before or after the predicate, an observation sentence (or statement) results. For example, one might observe in a given piece that "tone X has pitch" or that "note Y is louder than note Z," or that "the dyad consisting of tones P and Q forms a time-interval that matches the time-interval formed by the dyad consisting of tones R and S".

The predicates embodied in phrases such as 'has pitch" or "is louder than" are considered to be undefined. As such, they constitute a portion of the set of "undefined concepts." or "elementary categories" of the analysis. Each undefined predicate can be considered to embody a "meaning" that might be more or less elusive depending on how the predicate is employed. The potential elusiveness of a given predicate's meaning constitutes a threat to the intersubjectivity of its usage. Though there is no way to guarantee that another person will understand one's predicates in precisely the way that the analyst might desire, one can attempt to ward off misunderstanding to a certain extent by specifying certain permutational properties of given predicates, that is, the ways in which observables can be arranged around the predicates. For example, I can state that, according to my understanding, the predicate "is higher (in pitch) than" is irreflexive, antisymmetric and transitive : transitive, because if tone X is higher (in pitch) than tone Y, and tone Y is higher in pitch than tone Z, then it follows that tone X is higher than tone Z ; asymmetric, because if tone X is higher than tone Y, then tone Y cannot be higher than tone X ; and irreflexive, because tone X cannot be higher than itself, at least according to my understanding.[5] By specifying that a given predicate is irreflexive, asymmetric, and transitive, I am conveying certain aspects of what I mean by the predicate and how I intend to use it. In this way, I am trying to clarify my usage and thereby attempting to prevent misunderstanding.

The permutational properties of a predicate constitute its logical features. If one has made an observation statement, one can, on the basis of one's understanding of the predicate employed, specify a) what further observation

statements are necessarily entailed, b) what further observation statements would contradict it, and c) what further observation statements would be neither entailed nor contradictory, but rather merely consistent with it

Although such observation statements can have certain logical aspects, their predicates have in the past been termed "extra-logical" Calling them extra-logical serves to distinguish them from more narrowly logical terms such as the connectives "and," "or,' and "not." The latter serve a number of useful functions in an analysis. One of these functions is to combine with undefined predicates to form definitions of defined predicates. For example, one can define the notion of ' between-ness" in the following way : *If* tone X is higher in pitch than tone Y *and* tone Y is higher in pitch than tone Z, *then* the pitch of tone Y is between the pitches of tones X and Z. The preceding definition is framed in terms of the logical connectives "if...then" and "and." (As logicians have shown, statements joined by "if...then" can be reduced to the same statements joined in specific arrangements by "and," "or," and "not.")[6]

The Principle of Parsimony.

The logical and extra-logical terms used in an anaylysis constitute its basis. By and large, one attempts to reduce the number and complexity of the predicates in a basis to a minimum. The principle invoked by methodologists in this situation is that of economy, parsimony, or "Ockham's razor." The desirability of economy seems often to have been concidered self-evident. Nevertheless, a number of reasons can be advanced for attempting to be parsimonious in one's choice of undefined predicates. First, all other things being equal, one increases the danger of losing intersubjectivity as one increases the number of one's undefined predicates. Secondly, each undefined predicate represents a risk, for its adoption involves the assumption that it captures a significant aspect of the observables. Thirdly, if one takes a given undefined predicate and defines another predicate in terms of it (as was done with "is higher (in pitch) than" and "has a pitch that is between the pitches of," above), the connections between the two predicates becomes clearer than they might have been if one had adopted both predicates as undefined.

Although it is generally considered desirable to reduce the numbrr and complexity of one's basic predicates as far as possible, to do so indefinitely would be self-stultifying. If there were no undefined extra-logical predicates at all, there could be no observation statements at all Furthermore, if in the case of music, for instance, one were to exclude all the predicates that refer to tone quality (or timbre), one would not be able to capture the ways in which

tone colour is patterned or structured in many pieces. Admittedly, in some works (e. g., the canons in J. S. Bach's *Musical Offering* and the fugues in the same composer's *Art of Fugue*), the timbral stucture is trivial insofar as the composer has specified no particular tone colours for the notes. However, in other pieces (e. g., instrumental arrangements of the same works that have been made by later writers), timbral structure can be quite complex.

One's choice of undefined predicates serves to determine the domain of discourse of one's analysis. For instance, one might analyse a piece from the point of view of loudness, or pitch, or both, or both in combination with other variables. In this way, one might evolve a "theory of pitch," "a theory of rhythm", etc. Furthermore, one might conceive of a theory of pitch that adopted "has pitch" as its only undefined extra-logical predicate. Though one would be able to specify which tones were identical with each other by virtue of having pitch or not having pitch, one would not be able to describe further pitch relations or any interval relations at all. In short, there is no single, "rock-bottom" list of undefined predicates that have to be adopted in a musical analysis.

In order to recapitulate and move on, one can note that in the sort of analytical situation which I am describing, there are three components : a set of observables, a set of undefined predicates that are both logical and extra-logical, and a set of abservation statements. The set of observables is considered to be "given" and, in the absence of the other two types of sets, unanalysed and uninterpreted. One can compare two analyses of the same set of observables with regard to the number and complexity of the undefined predicates that they invoke.[7] If their respective sets of undefined predicates define the same domain of discourse, they are commensurate. One can determine whether two analyses are commensurate by commparing the undefined predicates that they invoke. Indeed, one of the main reasons why parsimony can be considered desirable is that one can readily determine what domain of discourse is specified by a parsimonious analysis and determine whether two such analyses are commensurate. In general, if two sets of undefined predicates are commensurate and equally numerous and complex, but one analysis includes observation statements not contained in the other but not *vice versa*, the former is to be preferred. The last assertion, which I will refine later, embodies the principle of what I term "relational multiplicity" or "relational richness".

Relational Richness.

From the point of view of the sort of analysis in which I am engaged, the ultimate goal of analysis is to add to one's knowledge of the observables. In

general, as the number of observation statements increases, one's knowledge increases as well. Another way of stating this idea is to say that as the number of observation statements increases, so too does the number of relations that one can assert among the observables. In this way, one is encouraged to observe an injunction that has been articulated in various informal ways by commentators on problems of method, namely, "Connect, always connect".[8] Furthermore, by jointly reducing one's undefined predicates to a minimum number or degree of complexity and increasing the number of one's observation statements to a maximum, one can realize the goal of "simplicity in complexity" or "elegance" that certain writers on questions of method have advocated informally.[9]

When one compares the permutational properties of various predicates, one finds that different predicates give rise to different degrees of relational richness, other things being equal. For example, one can demonstrate that the most fertile sort of relationship is that of identity. Since the predicate "is identical with (i.e., in a single respect such as pitch or loudness)" is reflexive, symmetric, and transitive, it can give rise to n^2 relations or observation statements when applied to n observed entities. By contrast, the predicate "is precisely identical with (i.e., in all conceivable respects)" can give rise to only n relations or observation statements, because it is merely reflexive.[10] In general, identity is the most fertile sort of relationship, What this means for analysis is that one generally attempts to discern identities and other relatively fertile relations wherever feasible, for doing so tends to increase relational richness. In doing so, however, one need not take a relation such as "is identical with (in some respect)" as primitive or undefined. Identity relations can emerge as special cases of matching relations, which in turn can emerge as negations of difference relations, which in turn can emerge as disjunctions of sequential relations.[11]

Restricted Relational Richness.

Just as one might be tempted to multiply undefined predicates needlessly, one might, because of the injunction to maximize relational richness, be motivated to multiply relations or observation statements beyond a point where one is adding to knowledge. For that reason, I have specified the notion of "restricted relational richness" in order to prevent a madcap proliferation of observation statements First, I exclude the positing of absolutely unfalsifiable or tautological relations such as "matches in pitch or differs In pitch from." If the tones being observed have pitch, the predicate "matches in pitch or

differs in pitch from" is trivially applicable to any pair of them. Such statements, which involve complementary relationships joined by the connective "or," are excluded because they add nothing to one's knowledge of the observables. Secondly, I exclude the joint positing of redundant relationships such as "is higher in pitch than or lower in pitch than" and "is different in pitch from". Positing one of these relations might add to one's knowledge, but, since both predicates have the same meaning, positing a second such relationship adds nothing to one's knowledge. Similarly, I exclude the joint positing of a relationship and its opposite in a fashion such as the following : "tone X is higher than tone Y" and "tone Y is lower than tone X." Such pairs of relations are also tautological or redundant.

Concluding Remarks.

If relational richness is a main aim in an analysis, there are certain resluts, it becomes possible to compare analyses. Frequently, analyse appear to be justified on the mere grounds that they do not contradict the observables or that they provide an "interesting perspective" on the observables.[12] According to the method outlined here, one can distinguish between analyses with regard to their respective domains of discourse and their respective degrees of relational richness. Indeed, a high degree of relational richness in an interpretation represents a high degree of what has been described informally in the past as simplicity. Though it might seem paradoxical, a relatively large number of relations in an analysis betokens a relative simplicity in the analysis. This point becomes clear if one observes the simplifying effect that a positing of identities and isomorphisms has on an analysis.[13] Secondly, if one adopts a method that aims to increase relational richness in an analysis, one can compare sets of observables, for example, actual pieces of music, with regard to their relative amounts of simplicity. As one turns from a comparison of analyses to a comparison of pieces, one still finds that a relatively large number of relations corresponds – paradoxically – to a relatively high degree of simplicity. For example, one tends to find, by and large, more relations in relatively "popular' forms than in relatively "serious" or "elite" forms, all other things being equal And one would generally find more relations in an early work of Mozart than in a late work. And so forth.[14] In this way, one arrives at my final point which is that though one might assert a greater amount of aesthetic value to an *analysis* by virtue of its containing a higher degree of relational richness, such a procedure would be rather perilous with regard to actual *pieces*, for it would seem that—for some people at least – more

highly valued music is music that is less rich in relations. However, one cannot make a serious assertion about the relative amounts of relational richness in two pieces or repertoires unless one has analysed them in terms of a single set of undefined concepts and assessed their respective degrees of restricted relational richness.

Notes and References

* This is a revised version of a paper read at the annual meeting of the American Society for Aesthetics in Storrs, Connecticut, March, 1985 I would like to thank Robert Cantrick for his many helpful remarks on earlier versions of this article.

1. Jay Rahn, *A Theory for All Music : Problems and Solutions in the Analysis of Non-Western Forms*, Toronto, University of Toronto Press, © 1983.

2. Nelson Goodman, *The Structure of Appearance*, 2nd ed., New York, Bobbs-Merrill, 1966.

3. Benjamin Boretz, "Meta-Variations : Studies in the Foundations of Musical Thought (1)," *Perspectives of New Music*, vol. 8, no. 1, Fall-Winter, 1969, pp. 1–74 ; "Sketch of a Musical System (Meta-Variations, part II)," *Perspectives of New Music*, vol. 8, no. 2, Spring-Summer, 1970, pp, 49-111 ; "The Construction of Musical Syntax (I) (Meta-Variations, Part IIIa)," *Perspectives of New Music*, vol, 9, no. 1, Fall-Winter, 1970, pp. 23-42 ; "Musical Syntax (II) (Meta-Variations, Part IIIb)," *Perspectives of New Music*, vol. 10, no. 1, Fall-Winter, 1971 pp. 232-70 ; "Meta-Variations, part IV : Analytic Fallout (I)," *Perspectives of New Music*, vol, 11, no. 1, Fall-Winter, 1972, pp. 146-223.

4. Note that this list is slightly different from the primitives discussed in my *Theory for All Music*, pp. 43-76. Observe further that one can remove the problematic notion of a "tone" by regarding "a pitch" and "a moment" as qualia and positing their "togetherness" as "a tone." Thereupon, "a note" can be regarded as a symbol of "a tone" or "some tones." See, Goodman, *op. cit*, pp. 209-27 ; Sir Alfred J. Ayre, *Philosophy in the Twentieth Century*, New York, Random House, © 1982, pp 256-58 ; and Goodman's chapter on

the "Theory of Notation" in his *Languages of Art : An Approach to a Theory of Symbols*, Indianapolis, Bobbs-Merrill, © 1968, pp. 127-76.

5. On various characterizations of relations, see a standard introductory text on logic or mathematics such as patrick Suppes *Introduction to Logic*, New York, Van Nostrand, 1957, pp. 208-26.

6. For a definition of "if...then" in terms of "and," "or" and "not," see Bertrand Russell, *Introduction to Mathematical Philosophy*, London, G. Allen and Unwin, 1919, p. 147. Note that one can reduce one's undefined logical connectives to a single primitive, namely, "is incompatible with." Cf. on this point, Russell, *op. cit.*, p. 148.

7. On ways of assessing the complexity of a primitive, see Goodman, *op. Cit.*, pp 66-75.

8. Cf, Arthur Krestler, *The Act of Creation*, New York, Macmillan. 1964.

9. See, for example, Paul Dirac's radical comments in "The Evolution of the Physicist's Picture of Nature," *Scientific American*, vol. 208, no. 5, May, 1963, pp 47-53, p. 47. Note that the formulation of restricted relational richness, below, represents a refinement of the methodolgical injunction to "establish the greatest number of similarities among the values and relationships by which the observables are interpreted" which is proposed in my *Theory for All Music*, p. 51. As will be seen below, similarities enjoy a privileged status according to the criterion of restricted relational richness, but one need not exclude diversities.

10. In general, n entities that are identical give rise to $n^2=1, 4, 9, 16, 25,...$ relations of identity, whereas n entities that are precisely identical (i e, only identical to themselves) give rise to only $n=1, 2, 3, 4, 5,...$ relations of precise identity.

11. If X matches Y, and matches Z, X may or may not match Z. If, in fact, X matches Z, then X, Y and Z are identical. If X matches Y, then X is not different from Y. With regard to pitch, one can say that if X is different from Y, then either X, is higher than Y or Y is higher than X. ("Is higher than" is considered to be a sequential relation, i.e., a relation that is irreflexive, symmetric and transitive : other sequential relations are "is before" and " is louder than").

12. The point of view adopted here is, thus, relatively demarcationist, in contrast with methodological relativism of the sort represented by Paul Feyeradend's *Against Method*, London, NLB, 1975.

13. Relational richness as a methodological criterion can serve to replace "simplicity." For example, a classic case where the latter notion, which has resisted analysis, has been employed can be accounted for according to relational richness, namely, the problem of explaining the desirability scientists find in drawinag straight line through three points, X, Y and Z, which might represent three measurements, rather than an arbitrary curve. If one draws a straight line rather than an arbitrary curve, one is implicitly asserting a number of identity relations between the observables, *viz.*, the slope of XY equals the slope of YZ, the slope of XY equals the slope of XZ, and the slope of YZ equals the slope of XZ. If one draws an arbitrary curve through three points, one loses all the relations of what one might call "slope-identity." For a survey of this classic problem in methodology, see Carl G. Hempel, *Philosophy of Natural Science*, Englewood Cliffs, N. J., Prentice-Hall, © 1966, pp. 40-45.

14. On the idea that relatively popular forms of music have been simpler than relatively "elite" or "serious" forms of music, see Gaynor Jones and Jay Rahn, "Definitions of Popular Music: Recycled," *Journal of Aesthetic Education*, vol. 11, no. 4, October, 1977, pp. 79-92 Note, however, that minimalist music of the sort produced by Steve Reich is anomolously very rich in relations, if only by virtue of the great numbers of identity relations that result from its extreme repetitiveness.

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